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CODING AND QUANTIZATION LOSSES FOR THE COHERENT WHITE GAUSSIAN CHANNEL

Boon J. Rit Nualprasert

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THESIS

CODING AND QUANTIZATION LOSSES FOR THE COHERENT WHITE GAUSSIAN CHANNEL

by

Boon J. Rit Nualprasert

Thesis Advisor: J. Geist

December 1972

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Coding and Quantization Losses

For the Coherent White Gaussian Channel

by

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Lieutenant Commander, Royal Thai Navy
B.Sc., Royal Thai Naval Academy, 1957

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

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ABSTRACT

Communication of binary data over an additive white Gaussian channel by means of coded transmission over coherent phase-shift keyed modems is considered. It is found that for high rate codes and coarse receiver output quantization, the minimum value of E_b/N_o required for reliable communication is significantly higher than the Shannon limit of -1.6 dB. for the raw channel. This loss of efficiency is investigated quantitatively as a function of code rate and coarseness of quantization, and shown to be negligible for low rate code with fine quantization. Similar results are presented showing the minimum E_b/N_o ratio for operation in the region where a sequential decoder performs finite average computation. In connection with this, tables and curves are given which show the value of the Pareto parameter ρ as a function of E_b/N_o.

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I. INTRODUCTION

A. CAPACITY OF THE ADDITIVE WHITE GAUSSIAN NOISE CHANNEL

Shannon's formula [Ref. 1] for the capacity of the band-limited additive white Gaussian noise (AWGN) channel, shown in the communication scheme of Figure 1 is

$$C = \frac{1}{2}\log_2\left(1 + \frac{P}{N_o W}\right) \qquad \text{bits/baud}$$

$$\text{or} \quad C = W\log_2\left(1 + \frac{P}{N_o W}\right) \qquad \text{bits/second}$$

where W is the bandwidth of channel, P is the average transmitted power, and $\frac{N_o}{2}$ is the noise power spectral density.

As W approaches infinity, i.e., the bandwidth is unlimited,

$$C = \lim_{W\to\infty} W\log_2\left(1 + \frac{P}{N_o W}\right)$$

Letting $W = \frac{1}{h}$

$$C = \lim_{h\to 0} \frac{1}{h}\log_2\left(1 + \frac{Ph}{N_o}\right)$$

$$= \lim_{h\to 0} \frac{1}{h} \cdot \frac{\ln(1 + Ph/N_o)}{\ln 2}$$

By L'Hospital rule

$$C = \lim_{h \to 0} \frac{\frac{P}{N_o + Ph}}{\ln 2}$$

$$C = \frac{P}{N_o \ln 2} \qquad \text{bits/second}$$

This is the capacity for the infinite bandwidth AWGN channel

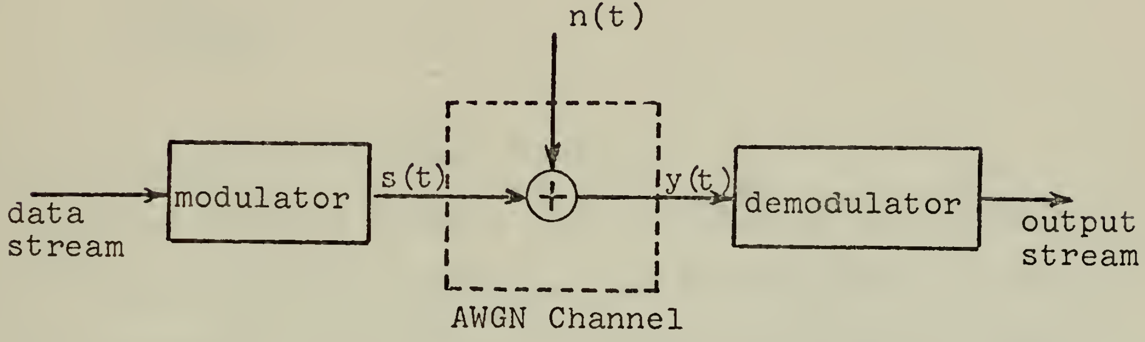


Figure 1. System block diagram

Let R denote the number of information bits to be transmitted per unit time (i.e., bits per second). Then, the energy per bit is $E_b = \frac{P}{R}$. Since C is the maximum number of bits per second which can be transmitted with arbitrarily small error probability, must have reliable communication requires that R be less than C, or equivalently

$$R = \frac{P}{E_b} < C = \frac{P}{N_o \ln 2}$$

$$\text{or} \quad \frac{E_b}{N_o} > \ln 2$$

$$\text{or} \quad \frac{E_b}{N_o} > 0.69 \quad \text{approximately}$$

Expressing the ratio in decibels, this becomes

$$\left(\frac{E_b}{N_o}\right) \text{dB} > -1.6$$

This is called the "Shannon limit."

The significance of this limit is that no matter what is in the boxes labeled "modulator" and "demodulator" in Figure 1, the probability of error can be made arbitrarily small only if E_b/N_o exceeds -1.6 dB; conversely, if E_b/N_o does exceed -1.6 dB, then there exists some "modulator" and "demodulator" to make the probability of error as small as desired.

B. DISCRETIZING THE CHANNEL

1. Bit-by-bit Antipodal PSK

One modulation for binary communication over the AWGN channel is the antipodal PSK system shown in Figure 2. The data stream is a sequence of ±1's so that over each baud of length T, the transmitted signal s(t) is a cosine of frequency ω_c and phase either 0 or π. Note that the energy

expended for information bit is

$$\int_0^T [s(t)]^2 \, dt = \int_0^T [\sqrt{\pm \frac{2E_b}{T}} \cos \omega_c]^2 \, dt$$

$$= E_b$$

where the simplifying assumption is made that a baud contains an integral number of cycles of the cosine. The demodulator in this system is a correlation detector which requires an oscillator synchronized with that at the transmitter; the system is thus called a coherent system. The integrator output at time T is tested against the threshold 0. If $Z(T) \geq 0$, the receiver decides that the data bit is +1, if $Z(T) < 0$, the decision is -1. The system operates identically over each baud, independent of all other bauds, and is therefore called a bit-by-bit system.

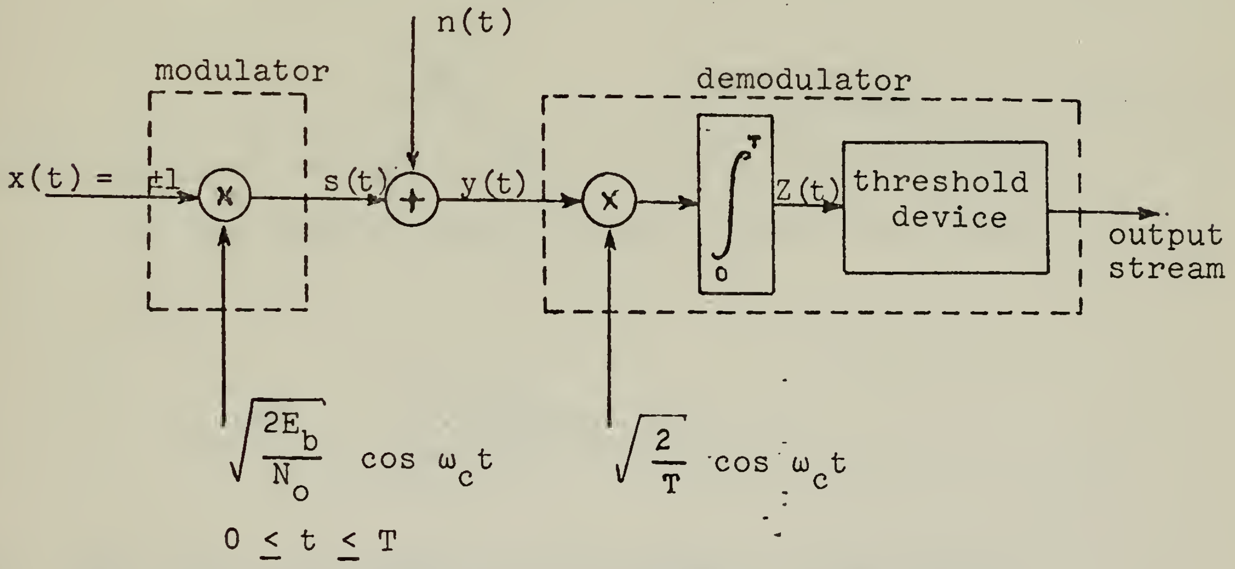


Figure 2. A model of bit-by-bit antipodal PSK system

Letting R again denote the data rate, the baud length must satisfy $T = \frac{1}{R}$.

Let the power spectral density function of white noise be $\frac{N_o}{2}$; thus its autocorrelation function is

$$R(\tau) = \mathcal{F}^{-1} \left(\frac{N_o}{2}\right)$$

$$= \frac{N_o}{2} \delta(\tau)$$

Consider in the case of signal input alone,

$$y(t) = \pm \sqrt{\frac{2E_b}{T}} \cos \omega_c t$$

from the integrator output at time T,

$$Z_s(T) = \frac{\pm 2 \sqrt{E_b}}{T} \int_0^T \cos^2 \omega_c t \, dt$$

$$= \pm \sqrt{E_b} \quad , \qquad \text{assuming that}$$

[0,T] contained integral number of cycles of the cosine.

In the case of noise present alone, the output is

$$Z_n(T) = \int_0^T \sqrt{\frac{2}{T}} n(t) \cos \omega_c t \, dt$$

So, when noise and signal are presented in the system

$$y(t) = s(t) + n(t)$$

$$Z(T) = Z_s(T) + Z_n(T)$$

$$= \pm\sqrt{E_b} + \int_0^T \sqrt{\frac{2}{T}}\, n(t) \cos \omega_c t \, dt$$

The quantity $Z_n(T)$ is a zero mean Gaussian random variable whose variance, $\sigma^2_{z_n}$, can be found as follows:

$$\sigma^2_{z_n} = E[(Z_n(T))^2]$$

$$= E\left[\left(\int_0^T \sqrt{\frac{2}{T}}\, n(t) \cos \omega_c t \, dt\right)^2\right]$$

$$= E\left[\frac{2}{T} \left(\int_0^T n(t) \cos \omega_c t \, dt\right)\left(\int_0^T n(s) \cos \omega_c s \, ds\right)\right]$$

$$= \frac{2}{T} \int_0^T \int_0^T E\left(n(t)\cdot n(s)\right) \cos \omega_c t \cdot \cos \omega_c s \, dt \, ds$$

But $R(\tau) = E[n(t)\cdot n(t-\tau)]$

$$\sigma^2_{z_n} = \frac{2}{T} \int_0^T \int_0^T R(t-s) \cos \omega_c t \cdot \cos \omega_c s \, dt \, ds$$

$$= \frac{2}{T} \int_0^T \int_0^T \frac{N_o}{2}\, \delta(t-s) \cos \omega_c t \cdot \cos \omega_c s \, dt \, ds$$

$$= \frac{2}{T} \int_0^T \frac{N_o}{2} \cos^2 \omega_c t \, dt$$

$$= \frac{N_o}{2}$$

The conditional probability density function of Z(T) given the data bit can be represented by Figure 3.

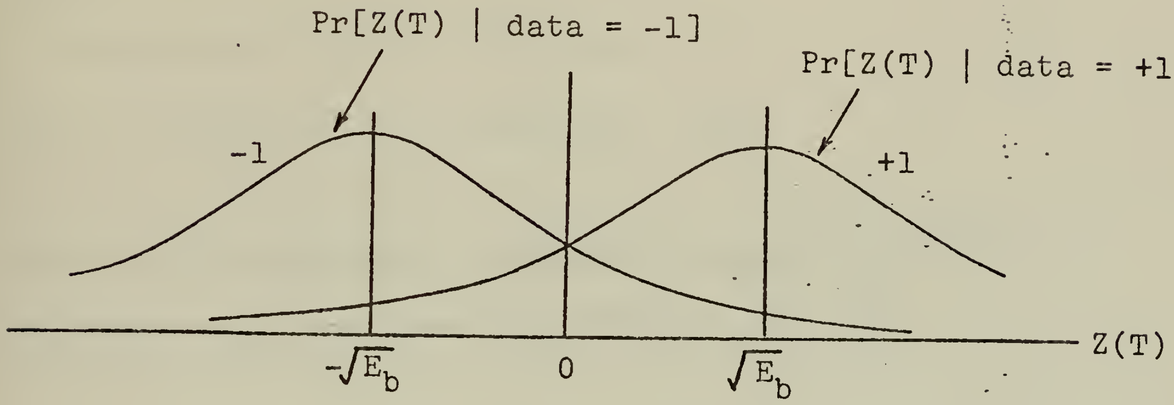


Figure 3. Conditional probability density function of Z(T) in the case of a positive input of amplitude 1, and in that of a negative input of amplitude -1.

Then, the probability of error, Pe, of the output stream after the threshold device can be found by

$$\begin{aligned} Pe &= Pr[(Z(T) \geq 0) \mid \text{data bit} = -1] \cdot Pr[\text{data bit} = -1] \\ &+ Pr[(Z(T) < 0) \mid \text{data bit} = +1] \cdot Pr[\text{data bit} = +1] \end{aligned}$$

Assuming equally likely data,

$$Pr[\text{data} = -1] = Pr[\text{data} = +1] = \frac{1}{2}$$

By symmetry

$$\Pr[Z(T) \geq 0 \mid \text{data bit} = -1] = \Pr[(Z(T) < 0 \mid \text{data bit} = +1]$$

thus

$$Pe = \Pr[(Z(T) \geq 0) \mid \text{data bit} = -1]$$

or $$Pe = \Pr[(Z(T) < 0) \mid \text{data bit} = +1]$$

Since this conditional probability density function is Gaussian with zero mean and variance $\sigma^2 = \frac{N_o}{2}$

$$Pe = \frac{1}{2} - \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^{\sqrt{E_b}} e^{-(x^2/2\sigma^2)}\,dx$$

$$= \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{E_b}/\sigma} e^{-(y^2/2)}\,dy \quad ,$$

where $y = \frac{x}{\sigma}$

$$= \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{(2E_b/N_o)}} e^{-(y^2/2)}\,dy$$

$$= Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

The probability of error is shown by the shaded area of the standard normal distribution function in Figure 4, where

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{-(y^2/2)}$$

In Figure 5 the curve of probability of error versus E_b/N_o in decibel is shown, along with the Shannon limit.

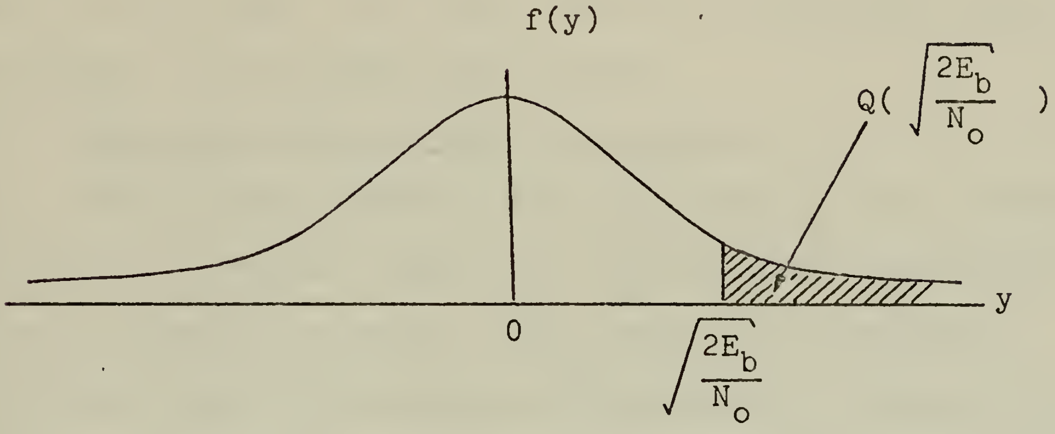


Figure 4. Probability of error. $Pe = Q(\sqrt{\frac{2E_b}{N_o}})$

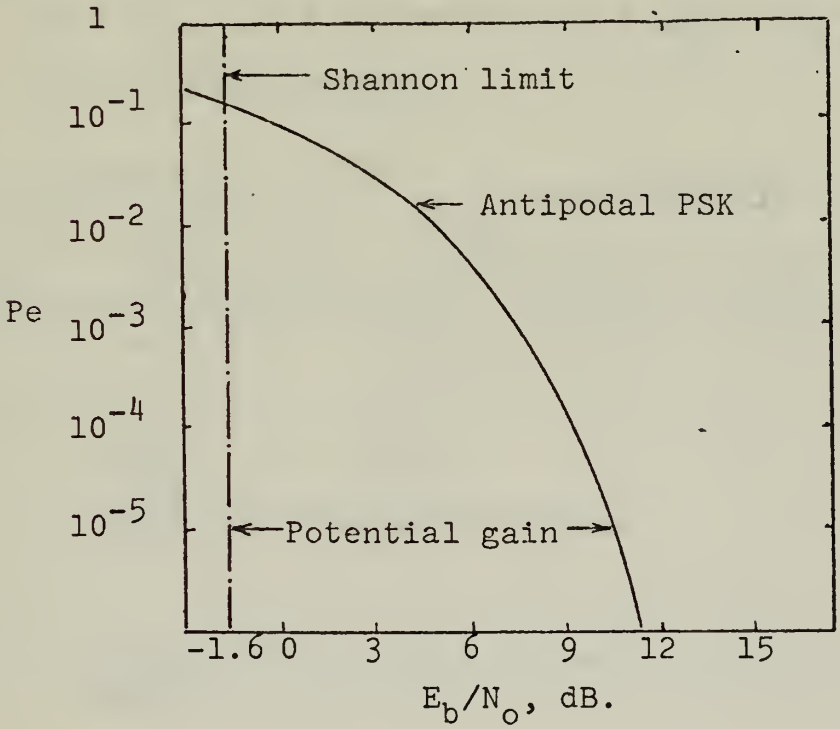


Figure 5. Probability of error as a function of SNR.

It can be seen that, for example, to obtain Pe of 10^{-5} with this scheme, $(E_b/N_o)_{dB}$ must be 9.6 dB. On the other hand, the capacity theorem says that there is some system with essentially zero error probability whenever E_b/N_o exceed -1.6 dB [Ref. 2]. Thus, there is a potential gain of 11.2 dB over the antipodal PSK system.

2. Antipodal PSK with Rate 1/2 Coding

Part of the potential gain indicated in the previous section can be realized when the somewhat more complicated system of Figure 6 is used. This system still employs a coherent antipodal PSK modulation/demodulation scheme, but it differs from the previous system (Figure 5) in two important respects: (1) more bits are transmitted per unit time than appear as data bits (in this case, twice as many); and (2) the bits transmitted at any time depend not only on the present data bit, but also on some past data bits (thus this is not a bit-by-bit system).

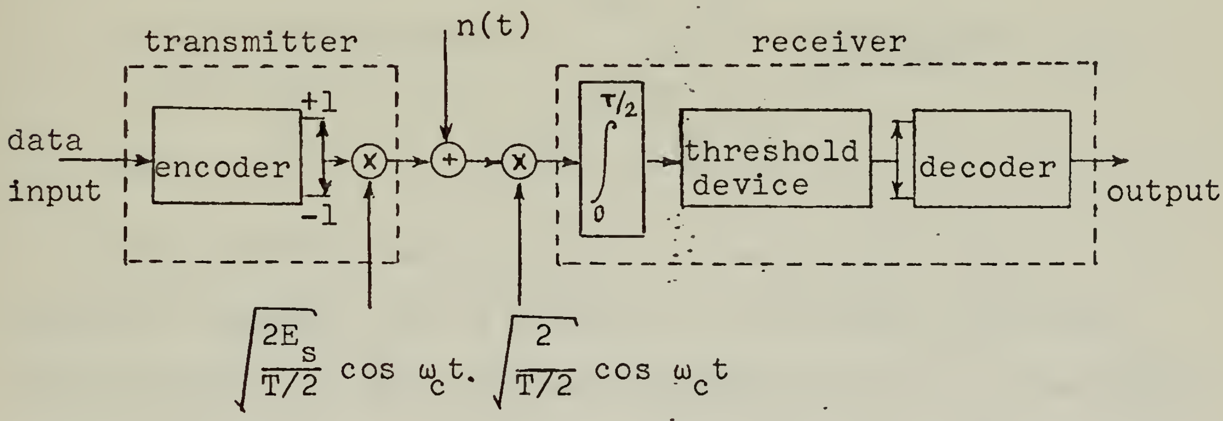
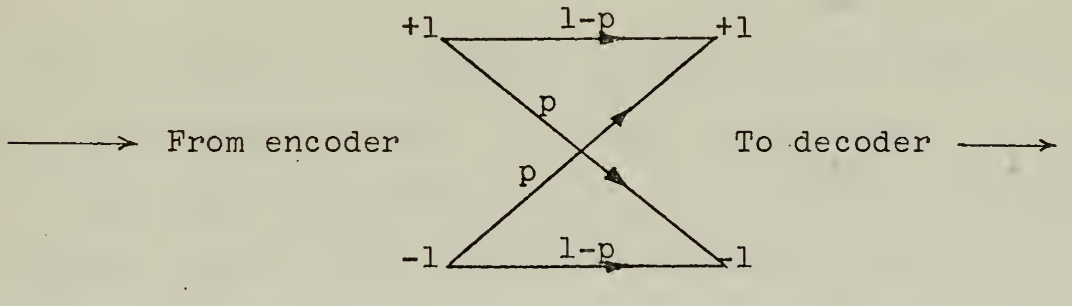


Figure 6. A model of antipodal PSK with rate 1/2 coding system.

Let data stream input be 1 bit for each T second interval, then, for code rate, R = 1/2, (R is defined to be the number of information bits per transmitted bit), the output from encoder will be 2 bits for each T second.

Let E_s be the energy expended per transmitted bit then $E_s = (1/2)E_b$. Assume that the use of the channel is as follows: each bit is transmitted by binary antipodal signaling. At the receiver, the best guess is made for each bit and +1 or -1 is the output, accordingly. Thus, from the output of the encoder to the input of the decoder, the channel can be though of as a discrete system with binary input and binary output acting as illustrated graphically below, where p is the error probability for bit-by-bit binary antipodal signaling. According to the solution in bit-by-bit antipodal PSK found previously, $p = Q(\frac{2E_s}{N_o})$



This discrete channel, which is called the binary symmetric channel, has a capacity, C, with the following significance: if the code rate R satisfied R < C, then there exists a code (i.e., an encoder-decoder pair) such that the error probability can be made as small as desired.

The capacity of the binary symmetric channel is given by

$$C = 1 - H(p) \qquad \text{bit/use of channel.}$$

where $H(p) = -p \log_2 p - (1-p)\log_2 (1-p)$

$$p = Q\left(\sqrt{\frac{2E_s}{N_o}}\right)$$

Thus for reliable communication in this system, it is necessary that

$$R = \frac{1}{2} < C = 1 - H\left[Q\left(\sqrt{\frac{2E_s}{N_o}}\right)\right]$$

$$= 1 - H\left[Q\left(\sqrt{\frac{2\cdot E_b/2}{N_o}}\right)\right]$$

$$\text{or } \left(\frac{E_b}{N_o}\right)_{\text{dB}} > 1.8 \text{ dB.}$$

When this condition is satisfied, there is some choice of encoder and decoder to make Pe. as small as desired.

Note that this minimum E_b/N_o is still 3.4 dB above the Shannon limit. This difference is due to use of $R = 1/2$ rather than a lower rate, and making hard decisions at the integrator output. In Chapter II, the minimum E_b/N_o is determined for various rates and output quantizations, and it is shown that for low-rate codes and fine output quantization, the Shannon limit is approached.

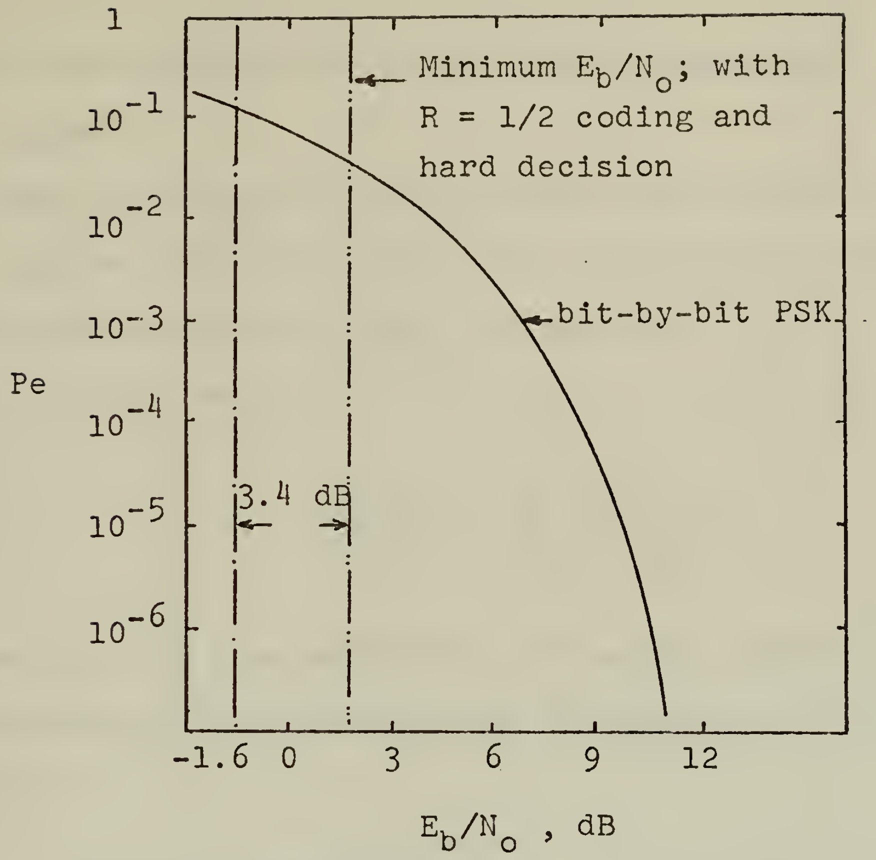


Figure 7. Minimum E_b/N_o, in dB, due to R = 1/2

C. CAPACITY OF DISCRETE CHANNELS

Consider a discrete memoryless channel (DMC) whose input alphabet X consists of the K integers, 0, 1, 2, ..., K-1, and whose output alphabet Y consists of the J integers, 0, 1, 2, ..., J-1. Using integers for input and output letters simplifies the notation somewhat in what follows and also reemphasizes the fact that the name given to input and output letters are of no concern whatsoever.

The channel is specified by a transition probability assignment, $P(j|k)$, given for $0 \leqq j \leqq J-1$, and $0 \leqq k \leqq K-1$,

by definition, $P(j|k)$ is the probability of receiving integer j given that integer k is the channel input. For example, see Figure 8.

Since the channel is memoryless, each output letter in the sequence depended only on the corresponding input, and the probability of an output sequence $Y = (y_1, y_2, \ldots y_N)$ given an input sequence $X = (x_1, x_2, \ldots x_N)$ is given by

$$P_N(Y|X) = \prod_{n=1}^{N} P(y_n|x_n)$$

Let $Q(k)$ be the probability of using integer k, then the average mutual information [Ref. 3] between the input and output is

$$I(X;Y) = \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} Q(k) \cdot P(j|k) \log_2 \frac{P(j|k)}{\sum_{i=0}^{K-1} Q(k)P(j|i)}$$

where, the probability of receiving integer j is written as $\sum_i Q(i)P(j|i)$

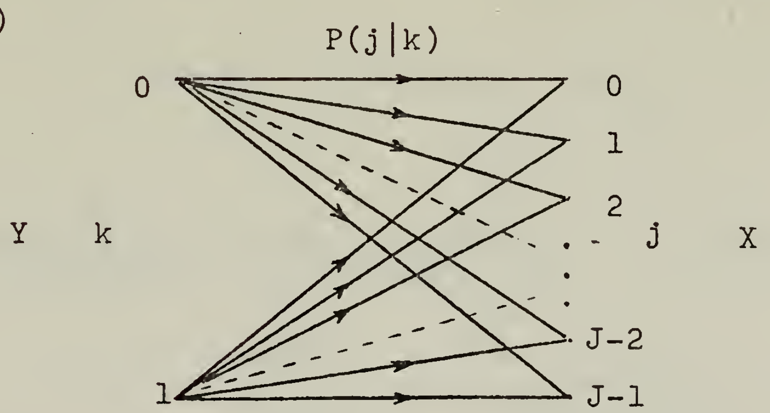


Figure 8. Transition probability for K = 2 input, and J output

The capacity of C of a discrete memoryless channel (DMC) is defined as the largest average mutual information I(X;Y) that can be transmitted over the channel in one use, maximized over all input probability assignments.

$$C = \max_{Q(0)\ \ldots\ Q(K-1)} \sum_{k,j} Q(k)\, P(j|k) \log_2 \frac{P(j|k)}{P(j)}$$

C is the function only of the channel. The calculation of C involves a maximization over K variable with both inequalities constrains, $Q(k) \geq 0$ and an equality constrains, $\sum Q(k) = 1$. The maximum value must exist since the function is continuous and the maximization is over a closed bounded region of vector space.

The channels of interest here all have two inputs and are symmetric, as shown in Figure 9, so that the capacity formula reduces to

$$C = \frac{1}{2} \sum_{j=0}^{J-1} [P(j|0) \log_2 \frac{P(j|0)}{P(j)} + P(j|1) \log_2 \frac{P(j|1)}{P(j)}]$$

$$P(j|0) = P(J-j-1|1) = A_{j+1}$$

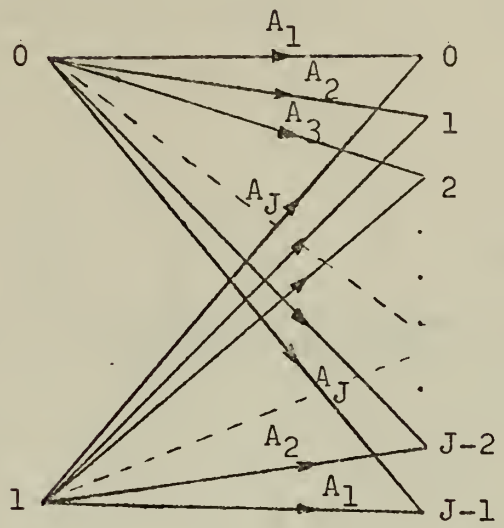
Then

$$C = A_1 \log_2 \frac{A_1}{(A_1+A_J)/2} + A_2 \log_2 \frac{A_2}{(A_2+A_{J-1})/2} + \ldots + A_3 \log_2 \frac{A_J}{(A_J+A_1)/2}$$

$$C = 1.0 + \sum_{i=1}^{J} A_i \log_2 \frac{A_i}{(A_i+A_{J-i+1})}$$

where A_i, i=1,2,...3; is the conditional probability of receiving output letter j given that input letter k is transmitted. All

details of modulator, channel and demodulator are compressed into the probabilities $\{A_i\}$



$$P(j=0) = \tfrac{1}{2}(A_1 + A_J)$$

$$P(j=1) = \tfrac{1}{2}(A_2 + A_{J-1})$$

$$P(j=J-1) = \tfrac{1}{2}(A_J + A_1)$$

Figure 9. Conditional probabilities $P(j|k)$ for J output and K = 2 input.
$P(j|0) = P(J-j-1|1) = A_{j+1}$

The amplitude quantizer may be one of the five shown in Figure 10, denoted J = 2,3,4,8 and 16[Ref. 4]. Considering for J = 2, K = 2

$$C = 1.0 + \sum_{i=1}^{J} A_i \log_2 \frac{A_i}{A_i + A_{J-i+1}}$$

$$= 1.0 + A_1 \log_2 A_1 + A_2 \log_2 A_2$$

$$= 1.0 + p\log_2 p + (1-p) \log_2 (1-p)$$

where $p = A_1$ $\quad 1-p = A_2$. $\quad p = Q\left(\sqrt{\frac{2E_s}{N_o}}\right)$

thus $C = 1.0 - H(p)$, channel capacity.

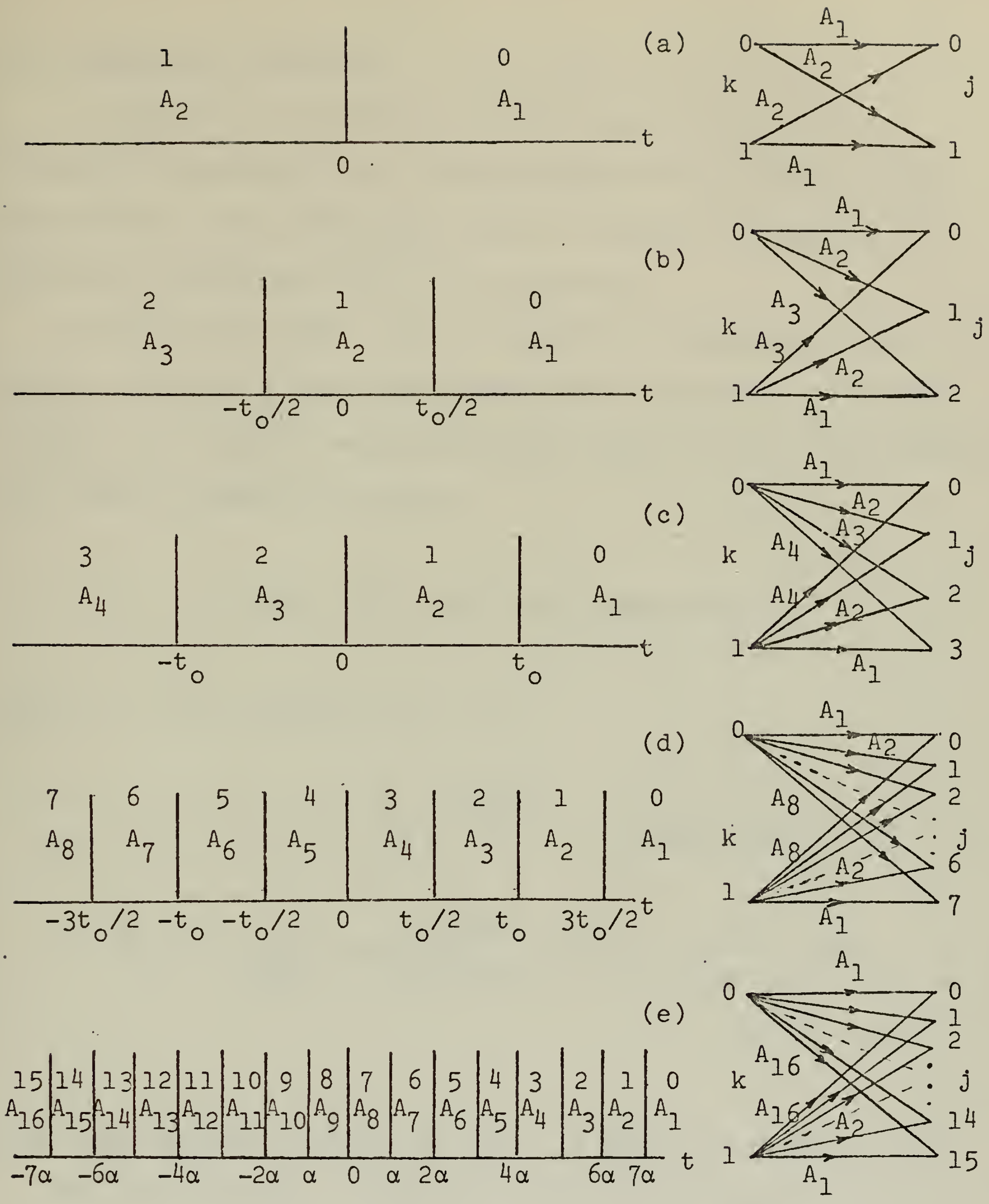


Fig. 10. Amplitude quantizers (a) J=2, hard decision, (b) J=3, binary-symmetric erasure, (c) J=4 with double null zone, (d) J=8, (e), J=10 with $\alpha = t_o/4$.

D. SEQUENTIAL DECODING

A sequential decoder is one which decodes by making tentative hypothesis as it goes along and by changing these hypothesis, when subsequent choices indicate an earlier incorrect hypothesis [Ref. 3, page 263].

From Gallager [Ref. 3], a property of sequential decoding systems is that the computation C required to decode a digit is a Pareto random variable; that is, its distribution, for large L assures the form

$$\Pr[C > L] = KL^{-\rho} \text{ as } L \text{ approaches } \infty$$

where ρ is the solution of $R = R_\rho$

$$R_\rho = \frac{E_o(\rho)}{\rho} \qquad \text{bits/channel use}$$

$$E_o(\rho) = \underset{Q(k)}{\text{Max}} \{-\log_2 \sum_{j=0}^{J-1} [\sum_{k=0}^{K-1} Q(k)\, P(j|k)^{1/1+\rho}]^{1+\rho}\}$$

$$0 < \rho < \infty$$

where the general formula of $E_o(\rho)$ is shown in Appendix A.

The computation a sequential decoder must perform to decode an information digit is a random variable, and R_ρ is the rate above which the ρ^{th} moment of this random variable fails to exist.When $\rho = 1$, the symbol R_{comp} is customary [Ref. 5]. In order to operate with finite average

computation, R must be less than R_{comp}. This condition can be translated into a minimum value of E_b/N_o in the same way that the condition of $R < C$ yield a minimum value of E_b/N_o. In Chapter III, minimum values of E_b/N_o are given for various rate and quantization levels.

II. CAPACITY LOSSES DUE TO CODING AND OUTPUT QUANTIZATION

From page 19, the channel capacity is

$$C = 1.0 + \sum_{i=1}^{J} A_i \log_2 \frac{A_i}{A_i + A_{J-i+1}}$$

Curves for various amplitude quantizers as shown in Figure 10 are presented in Figure 11. The curves are plotted for the value of E_b/N_o required to achieve $C = R$ versus R for $J = 2,3,4,8$ and 16, where $K = 2$.

Table I gives the value of E_b/N_o vs R for various values of J.

III. R_{comp} LOSSES DUE TO CODING AND OUTPUT QUANTIZATION

Curves of E_b/N_o versus R are presented in Figure 12 for various values of J to achieve $\rho = 1$, or $R_{comp} = R$, where $K = 2$.

Table II gives the value of E_b/N_o vs R. Here, as in Table I, R is treated as a continuous variable, although in practice R is usually the reciprocal of integer to simplify coder and decoder design.

IV. PERFORMANCE OF ρ VERSUS E_b/N_o FOR $R = R_\rho$

By solving $R = R_\rho = \frac{E_o(\rho)}{\rho}$, curves of ρ vs E_b/N_o for various values of R are presented in Figures 13, 14 and 15 for J = 2,4 and 8 respectively and are useful for design tradeoff.

It will be seen that, for example, in Figure 15 at $\rho = 1$ the value of E_b/N_o is 2.65 dB for R = 1/2 corresponded to Figure 12 at R = 1/2, and at $\rho = 0$ the value of E_b/N_o is -0.66 dB for R = 1/4 that is the same with the value of E_b/N_o in Figure 11 for J = 8, R = 1/4.

TABLE I

Table of (E_b/N_o)dB Required to Achieve C = R

R	J=2	J=3	J=4	J=8	J=16
3/4	3.01	2.34	2.17	1.76	1.66
2/3	2.51	1.75	1.56	1.18	1.09
1/2	1.77	0.90	0.65	-.30	0.22
1/3	1.21	0.28	-0.04	-.37	-.45
1/4	0.97	0.02	-.32	-.66	-.74
1/8	0.65	-.32	-.70	-1.04	-1.13
1/16	0.51	-.47	-.88	-1.22	-1.31
1/32	0.44	-.55	-.96	-1.30	-1.40
1/64	0.40	-.58	-1.00	-1.35	-1.44
1/128	0.39	-.60	-1.02	-1.37	-1.46

TABLE II

(E_b/N_o)dB Required to Achieve $\rho = 1$, $R_{comp} = R$

R	J=2	J=3	J=4	J=8	J=16
3/4	5.71	4.54	4.37	3.67	3.52
2/3	5.26	4.10	3.85	3.25	3.11
1/2	4.59	3.46	3.18	2.64	2.51
1/3	4.10	2.99	2.70	2.20	2.08
1/4	3.89	2.80	2.51	2.02	1.90
1/8	3.62	2.55	2.25	1.78	1.67
1/16	3.50	2.44	2.14	1.68	1.56
1/32	3.44	2.39	2.09	1.63	1.51
1/64	3.41	2.36	2.06	1.60	1.49
1/128	3.39	2.35	2.05	1.59	1.48

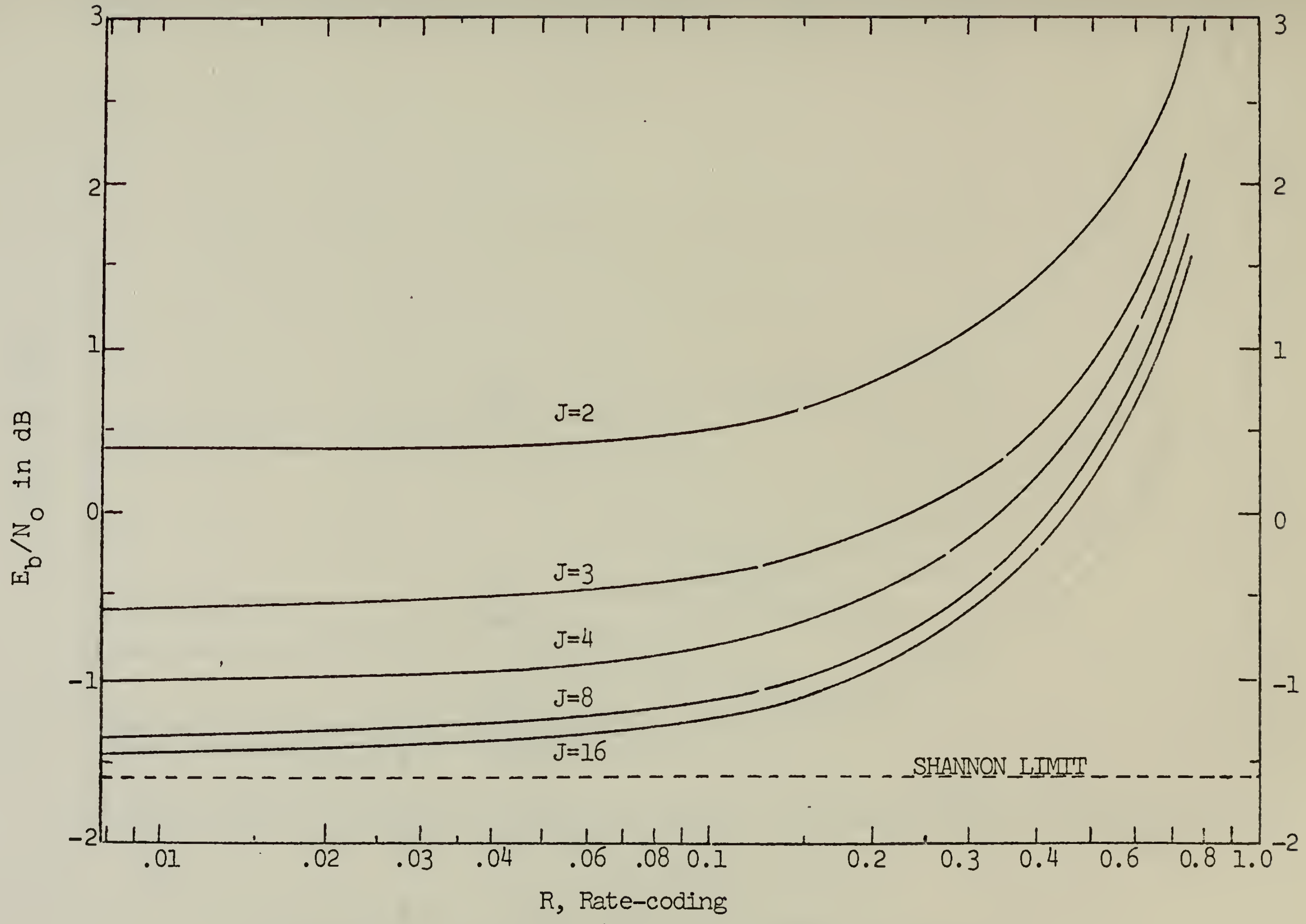


Figure 11. E_b/N_o required to achieve C = R.

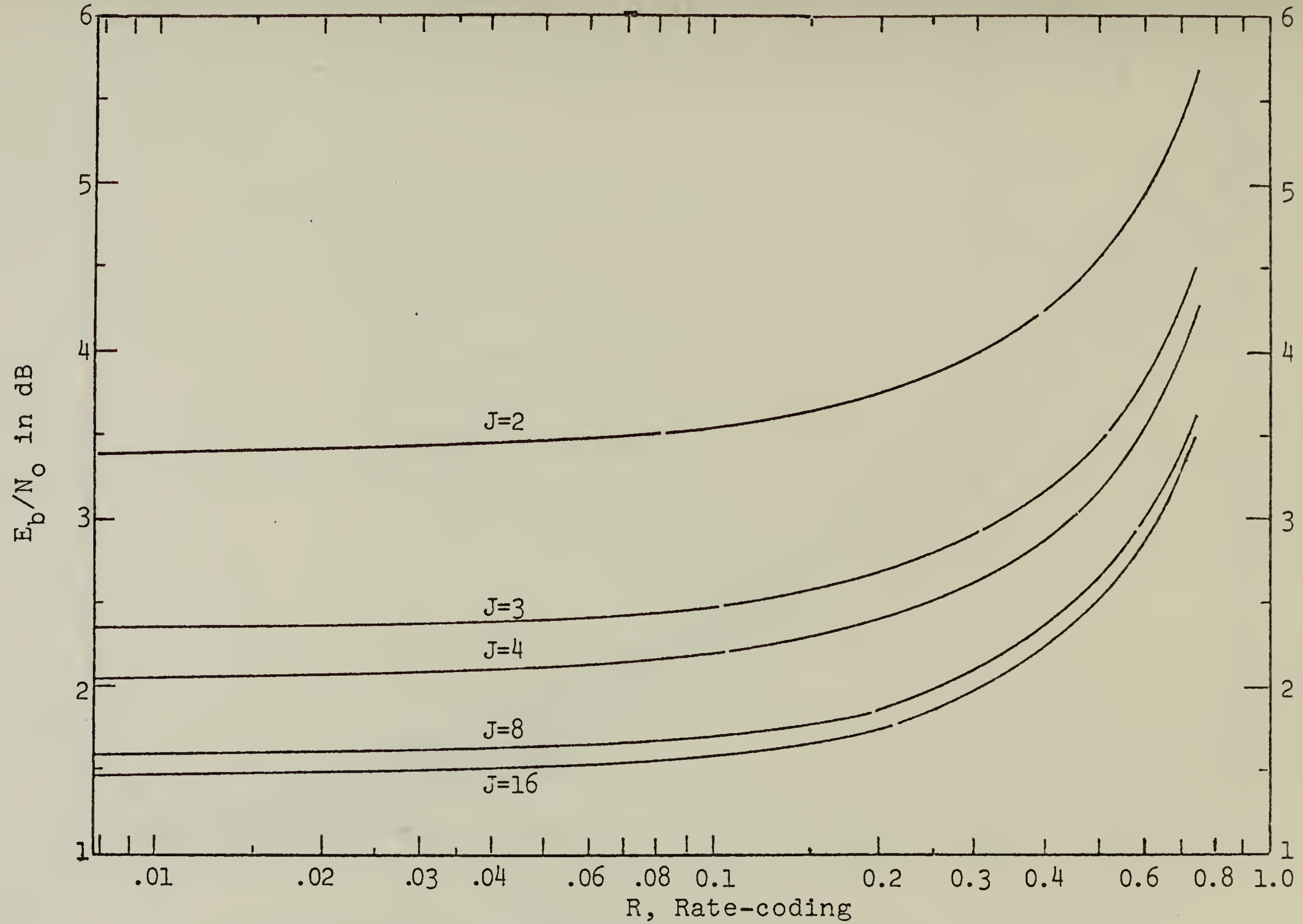


Figure 12. E_b/N_o required to achieve $R_{comp} = R$

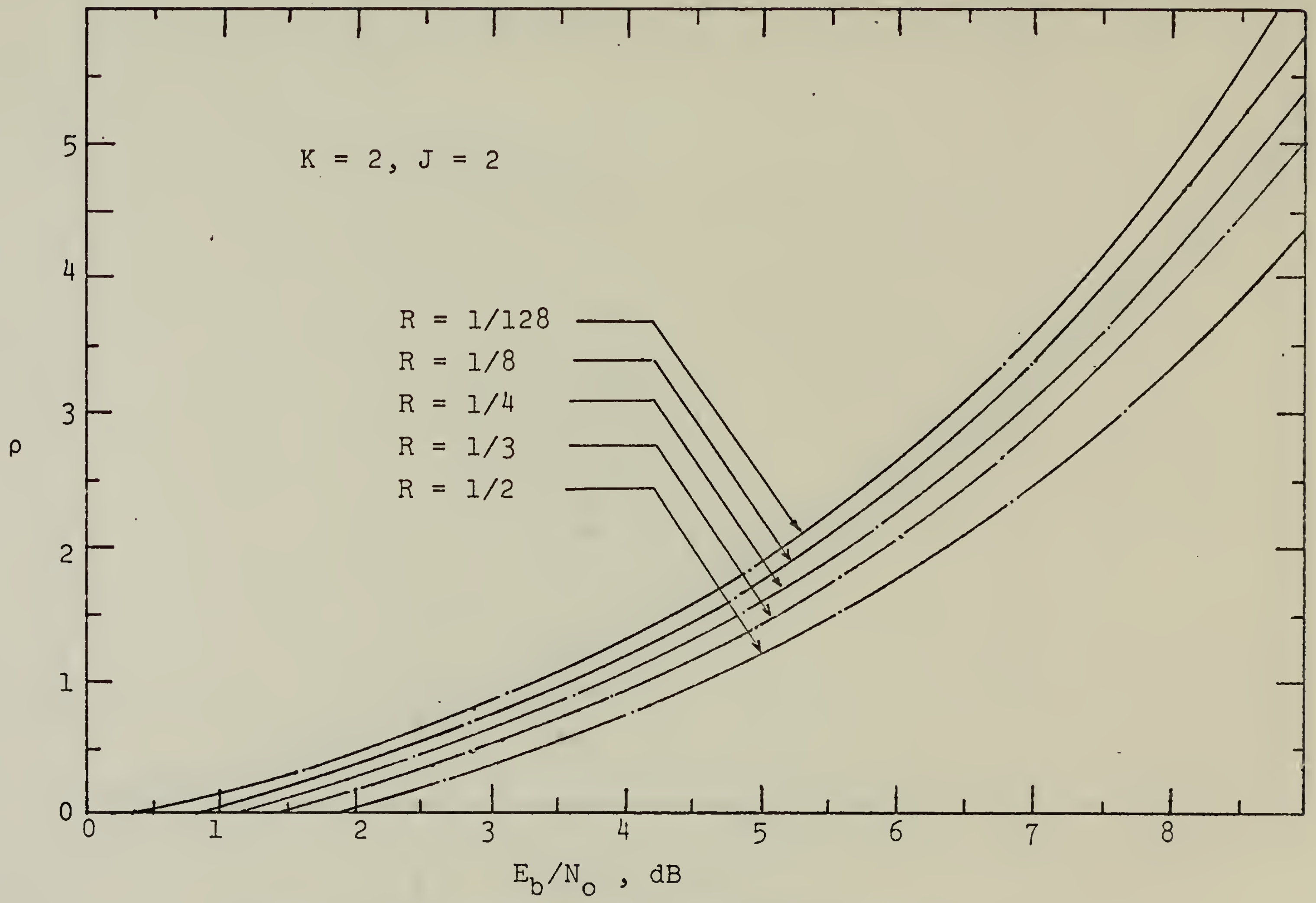


Figure 13. Dependence of ρ on E_b/N_o, dB.

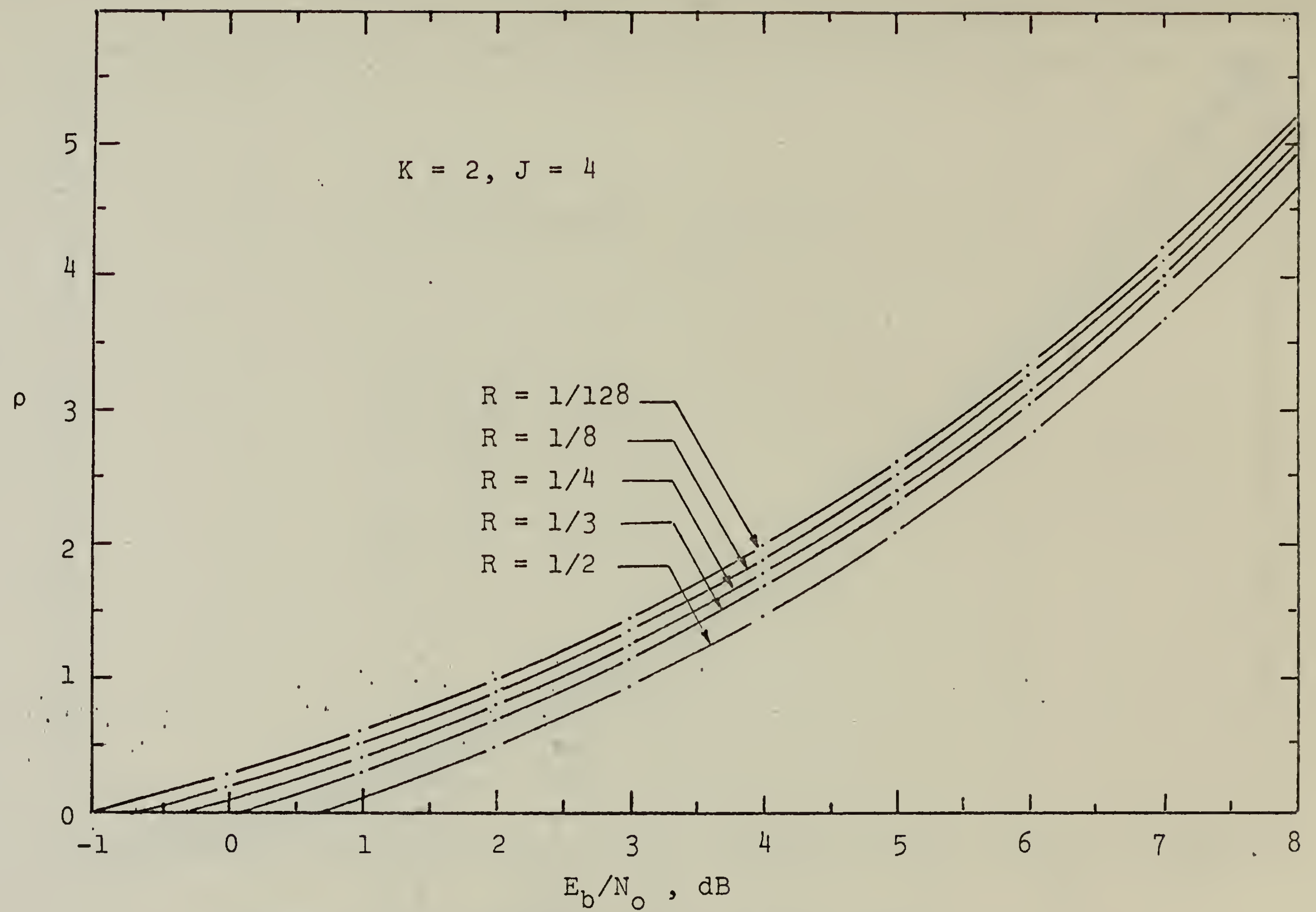


Figure 14. Dependence of ρ on E_b/N_o, dB.

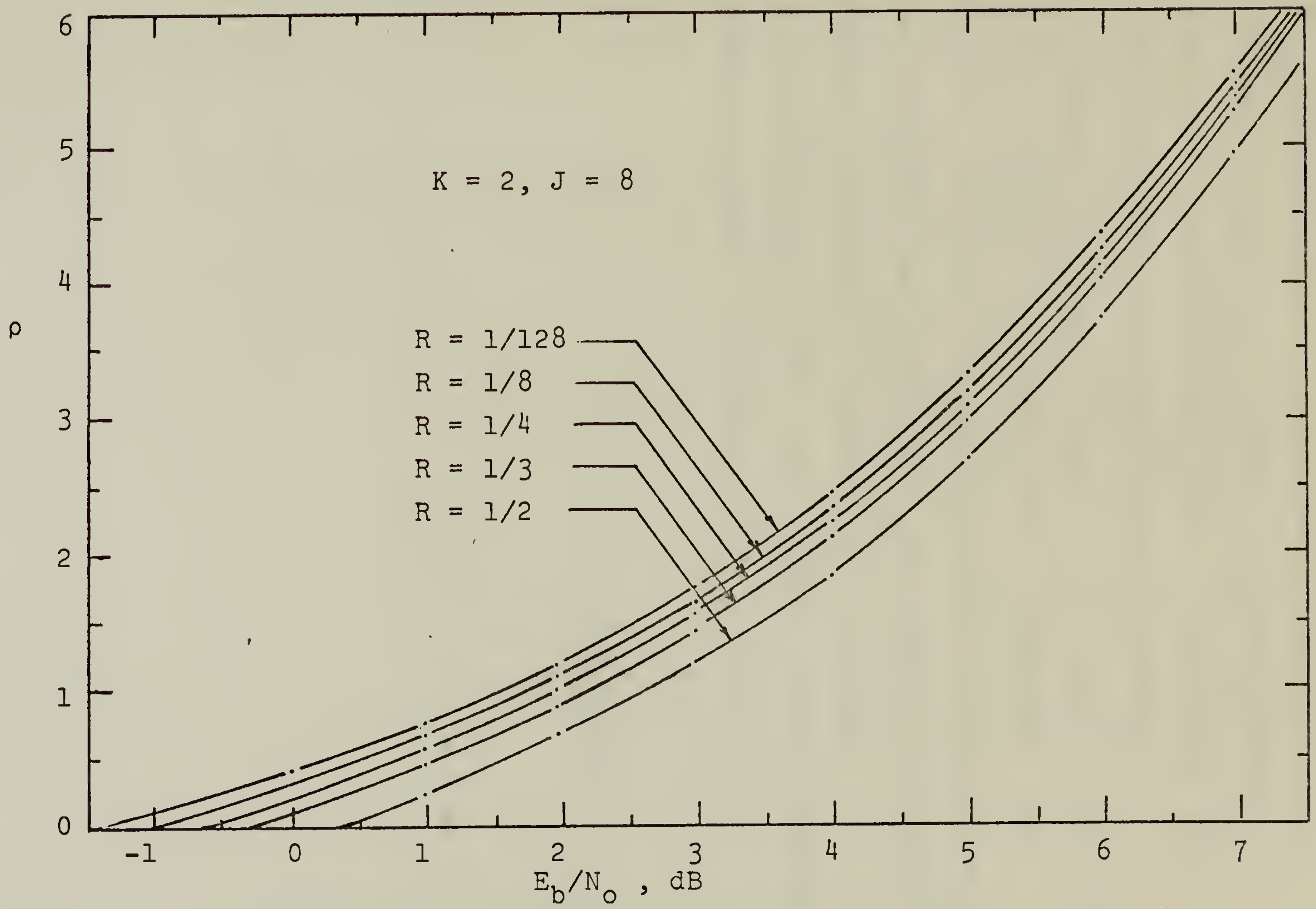


Figure 15. Dependence of ρ on E_b/N_o , dB.

V. SUMMARY AND CONCLUSION

The objective has been to present a unified treatment of efficiency losses due to coding and quantization in a PSK system with additive white noise. Performance curves and tables are given for useful design tradeoffs.

In Chapter I the basic concept of the Shannon limit with respect to antipodal bit-by-bit PSK is presented. In Chapter II, III, and IV, tables and curves showing expected system performance are presented. The expected performance figures were obtained by computer-aided analysis.

It is shown that in the limit of fine output quantization and low-rate coding the Shannon limit is approached.

APPENDIX A

GENERAL FORMULA OF $E_o(\rho)$ AND R_{comp}

From Gallager [Ref. 3] define for a discrete memoryless channel with K input symbols and J output symbols,

$$E_o(\rho) = \underset{Q}{\text{Max}} \{- \log_2 \sum_{j=0}^{J-1} [\sum_{k=0}^{K-1} Q(k)P(j|k)^{1/1+\rho}]^{1+\rho}\}$$

and
$$R_\rho = \frac{E_o(\rho)}{\rho}$$

Let $1+\rho = r$, $Q(k)_{max} = Q(k)$, $K = 2$

$$E_o(\rho) = - \log_2(Q(k))^r - \log_2 \sum_{j=0}^{J-1}[P(j|0)^{1/r}+P(j|1)^{1/r}]^r$$

$$= - \log_2(Q(k))^r - \log_2\{[P(0|0)^{1/r}+P(0|1)^{1/r}]^r$$

$$[P(1|0)^{1/r}+P(1|1)^{1/r}]^r$$

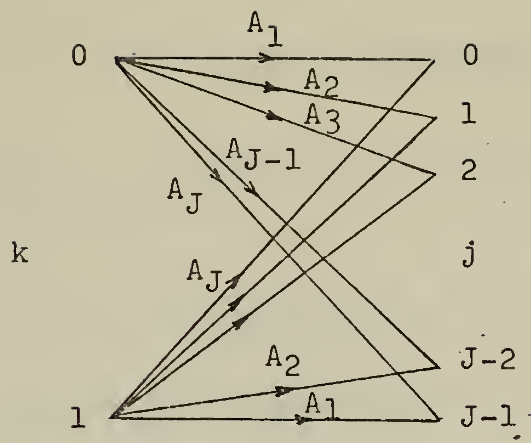
$$+ \ldots\ldots\ldots$$

$$+ [P(J-1|0)^{1/r}+P(J-1|1)^{1/r}]^r\}$$

$$E_o(\rho) = - \log_2(Q(k))^r - \log_2\{(A_1^{1/r}+A_J^{1/r})^r+(A_2^{1/r}+A_{J-1}^{1/r})^r+$$

$$\ldots\ldots\ldots +(A_J^{1/r}+A_1^{1/r})^r\}$$

where A_i, $i = 1,2,3,\ldots,J$ is the conditional probability of j given k as shown in the following figure for $K = 2$.



$$A_{j+1} = P(j|0)$$
$$= P(J-j-1|1)$$

$$E_o(\rho) = -\log_2 (Q(k)_{\max})^r - \log_2 \left[\sum_{i=1}^{J} (A_i^{1/r} + A_{J-i+1}^{1/r})^r\right] \quad \ldots A.1$$

For Q(k) maximum = $\frac{1}{2}$

$$R_{comp} = -2.0 \log_2 (\tfrac{1}{2}) - \log_2 \left[\sum_{i=1}^{J} (A_i^{1/2} + A_{J-i+1}^{1/2})^2\right]$$

$$= 2.0 - \log_2 [(A_1^{1/2} + A_J^{1/2})^2 + (A_2^{1/2} + A_{J-1}^{1/2})^2 + \ldots$$

$$+ (A_J^{1/2} + A_1^{1/2})^2]$$

$$R_{comp} = 2.0 - \log_2 \left[2 \sum_{i=1}^{J} A_i + 2 \sum_{i=1}^{J} (A_i A_{J-i+1})^{1/2}\right]$$

$$= 1.0 - \log_2 \left[1.0 + \sum_{i=1}^{J} (A_i A_{J-i+1})^{1/2}\right] \quad \ldots A.2$$

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13. ABSTRACT

Communication of binary data over an additive white Gaussian channel by means of coded transmission over coherent phase-shift keyed modems is considered. It is found that for high rate codes and coarse receiver output quantization, the minimum value of E_b/N_o required for reliable communication is significantly higher than the Shannon limit of -1.6 dB. for the raw channel. This loss of efficiency is investigated quantitatively as a function of code rate and coarseness of quantization, and shown to be negligible for low rate code with fine quantization. Similar results are presented showing the minimum E_b/N_o ratio for operation in the region where a sequential decoder performs finite average computation. In connection with this, tables and curves are given which show the value of the Pareto parameter ρ as a function of E_b/N_o.

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	ROLE	WT	ROLE	WT	ROLE	WT
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